

Segregation Processes

Elizabeth E. Bruch Robert D. Mare

CCPR-032-08

December 2008 Latest Revised: October 2007

> California Center for Population Research On-Line Working Paper Series

SEGREGATION PROCESSES*

Elizabeth E. Bruch University of Michigan

Robert D. Mare University of California -- Los Angeles

October 2007

*To appear in the *Oxford Handbook of Analytic Sociology*, edited by Peter Bearman and Peter Hedstrom. This research was supported by the Robert Wood Johnson Foundation, the National Science Foundation, and the Russell Sage Foundation.

SEGREGATION PROCESSES

INTRODUCTION

A pervasive property of social organization is that persons with similar social characteristics tend to cluster in close physical and social proximity. Racial and ethnic groups in the United States tend to reside in separate communities (Massey and Denton 1993). Throughout the world, men and women are segregated into distinct jobs and occupations (Charles and Grusky 2004). And small scale social units such as marriages and families tend, by processes of assortative mating, to bring together individuals with similar religion, education, age, social class background, and other traits (Kalmijn 1998). A central goal of sociological inquiry is to understand the segregation of people in physical and social space and the processes through which these patterns arise and persist. Segregation may occur through benign processes, such as the concentration of married people in small towns or babies in parks on sunny afternoons. But it may also reflect the workings of social hierarchies. For instance, disadvantaged ethnic minorities often live closest to areas of environmental pollution; neighborhoods reflect hierarchies of income and wealth; and the scarcity of women in executive positions results from the different behaviors of men and women and the different ways that men and women are treated. A common feature of segregation processes is that they are selfreinforcing. That is, the characteristics of settings across which groups are segregated are interdependent with the actions of individuals within the groups. If low rents draw poor people to less desirable neighborhoods, government may more easily dispose of waste in these already disadvantaged areas. This deters in-migration by more affluent individuals and reinforces neighborhood economic segregation. Segregation also tends to limit the choices available to individuals. For example, a white family who prefers to live in an integrated community may be

limited to either virtually all-white or all-black neighborhoods. Given a limited set of choices, they opt to remain with their own group, further sustaining segregation.

This essay is concerned with models of segregation processes and the methods and data for studying these processes empirically. We describe segregation in general terms and discuss the key parts of segregation processes: the actors, dimensions of clustering, and universe in which clustering occurs; focus on residential segregation and review measures, data, and models for investigating the dynamic interdependence of individual action and neighborhood characteristics; and briefly consider how multiple segregating processes may amplify or attenuate one another.

ANALYSIS OF SEGREGATION

Segregation is the nonrandom allocation of people who belong to different groups into social positions and the associated social and physical distances between groups. Segregation *per se*, is a static property of a population, whereas segregation *processes* are the actions that create and sustain segregation.

Single and Multiple Sets of Actors and Dimensions of Segregation

In principle, segregation may result from individuals' behaviors at a single level of social organization. For example, individuals may choose dwellings in a simple real estate market where the only other actors are buyers, sellers, and their neighbors. More realistically, however, individual actions are coordinated in a more complex way. Buyers, sellers, renters, landlords, agents, lenders, and politicians all affect real estate transactions. In the interests of presenting the current state of the art, we focus here mainly on segregating processes that are produced primarily though the behavior of individuals at one level of social organization (e.g., individuals

who decide where to move), but we recognize the importance other actors as well.

Social research typically examines segregation processes in only one social dimension or social context, but in fact people are segregated in multiple dimensions and contexts. For example, the same socioeconomic criteria by which children are distributed across neighborhoods may also funnel them into schools, peer groups, and eventual occupational trajectories. These contexts may overlap and amplify the inequality-producing effects of segregation in other environments. For example, residential segregation, political boundaries, and the structure of school taxation may affect human capital formation and lifelong inequalities (Benabou 1996; Durlauf 1996). In addition, a change in segregation along one dimension (e.g., poverty status) may exacerbate or mitigate segregation on another dimension (e.g., race) (Wilson 1987). Further, segregation occurs in contexts other than residential neighborhoods, including differential sorting in marriages, occupations, peer groups, lunchrooms, dormitories, prisons, church congregations, and even chat rooms in cyberspace. Winship and Tao (this volume) propose that time as well and physical or social space is yet another dimension along which segregation may occur. Variations in the models and methods discussed here for residential segregation are potentially useful in these other contexts as well.

The analysis of segregation consists of summarizing how a population that is differentiated into groups (e.g., economic strata, race, language, or gender) is distributed across social locations (e.g., neighborhoods, congregations, marriages, college dormitories, or jobs) and of discovering the processes that establish and maintain that distribution. The social groupings of interest usually reflect normative concerns with barriers between socioeconomic, ethnic, cultural, gender, and other groups. The social locations of interest also reflect normative concerns with segregation in work, residence, leisure, worship, and other contexts.

Segregation of Groups across Social Locations

We can classify the locations across which groups are segregated along two dimensions. First, segregation may be across groups of fixed or variable size. Examples of locations of relatively fixed size include marriages, jobs in a workplace, housing units in neighborhoods, and dinner seating at a wedding reception (assuming a fixed number of chairs per table). Locations of variable size include cliques in high school, church congregations, or picnickers on a beach. Second, segregation may involve the formation and dissolution of new locations, or entries and exits from enduring groups. New locations are formed when people marry, picnic on the beach, or form a friendship group. Often these locations are temporary and exits result in their dissolution. On the other hand, some locations endure beyond the lives of their incumbents. Enduring groups include work organizations, church congregations, or residential neighborhoods. Together these dimensions combine into four ideal-types: (1) fixed transitory, (2) fixed permanent, (3) variable transitory and (4) variable permanent. These types of locations vary in the segregation and segregation processes that they create. Segregation across transitory locations comprises not only the uneven distribution across these locations, but also differential participation. For example, the homogamy of marriages is affected not only by who marries whom given marriage, but also by who marries and which marriages persist (Schwartz and Mare 2003). In contrast, segregation across permanent locations usually comprises only the uneven distribution across locations. For example, residential neighborhoods are relatively permanent arrangements and all individuals live in some kind of neighborhood. The dynamics of segregation across fixed-size locations are, by definition, mediated by vacancies. One person cannot enter a neighborhood unless another person leaves. The pattern of segregation can only change by individuals' coordinated entries and exits from the neighborhood. In variable-sized

locations, in contrast, segregation may evolve through growth and decline as well as exchanges of dwellings between new and departing residents. Residential neighborhoods and job locations within firms have elements of all four locations, although in the short to medium term they are fixed permanent locations.

The Problem of Scale

Segregation is created and maintained through the interdependent actions of individuals. The mechanisms that link individual action and aggregate segregation may work on multiple scales (Levin 1992). For example, although segregation may occur across rectangular neighborhoods of 20 blocks, individuals may be aware of 1 or 2 block areas, or their overall metropolitan environments, or social networks that are only loosely tied to geography. In addition, behavioral regularities may vary across geographical scales, and thus micro-level processes may not aggregate in a simple way. For example, the factors governing individual residential preferences may vary across different levels of geography and individual actions may affect segregation at these geographic levels in different ways. A challenge to segregation research is to identify and account for these variations.

Social Interactions and Segregating Processes

Segregation processes result from interdependence between the actions of individuals and the characteristics of groups. Segregation may arise from a process in which individuals have preferences about a population statistic (for example, the proportion of white households in a neighborhood), and their own characteristics or actions also contribute to that statistic (Schelling 1971; 1978). (For discussion of the explanatory status of preferences, see Freese, this volume.) When a person leaves a neighborhood because she cannot tolerate its ethnic composition, she marginally changes the ethnic makeup of both the neighborhood that she vacates and also the

neighborhood into which she moves. This may induce other individuals to move because the relative attractiveness of neighborhoods has changed. The makeup of neighborhoods results from the individuals' past cumulative and interdependent choices. In the short run, individuals respond to their environments; in the longer run, individuals' responses change the environment for everyone. There may be no straightforward way to infer patterns of neighborhood segregation from the tolerance, intentions, or behaviors of individuals. Micro-level preferences and actions may generate a range of possible neighborhoods. Even a relatively tolerant population, for example, may collectively create highly segregated neighborhoods (Schelling 1971). Models that seek to explain such aggregate outcomes of interdependent behavior must represent feedback between individuals' actions and neighborhood characteristics.

Models for feedback effects can also describe segregation in other contexts. Occupations may "tip" from predominantly male to predominantly female (and *vice versa*). Although exogenous forces such as a shortage of men during times of war may effect massive shifts in the sex composition of jobs, changes in the sex composition of jobs may also be governed by endogenous processes (Reskin and Roos' 1990). An initial change toward feminization of an occupation may make it less attractive to men, which further lowers the number of men who work in that occupation, creating a cascade of female entrances and male exits from the position. Male-to-female turnover may be associated with a decline in the job's skill, status, and wages, which further affects the job's relative appeal (e.g., Kalleberg, Wallace, and Althauser 1981).

The creation of marriages also entails interdependent behavior. Hernes' (1972) examines the transition of individuals in a birth cohort from being single to married, and shows how the aggregate transformation of marital statuses results if the pressure to marry is

proportional to the proportion of those already married in the same cohort and the rate of marriage is proportional to this pressure. An extension of this model can incorporate the effects of not only the fraction of a cohort that is already married, but also the homogamy of existing marriages (on, for example, race). The marriage behavior of the cohort depends on both the race-specific proportions of persons who are already married, and also the normative climate for inter and intra-racial marriage established by those already married.

Interdependent actions also determine the specific marriages that occur through "twosided matching" (Roth and Sotomayor 1990). The preferences of one sex define the marriage opportunities of the other. A match only occurs if no man that the eligible woman prefers to her potential husband-to-be also prefers her to his current match. Changing even one person's preferences may affect the whole system of stable matches. Taken together, the diffusion of marital statuses through cohorts of men and women; the norms about whether, when, and whom to marry established by those who are already married; and the coordinated actions of the twosided marriage markets determine the numbers and types of marriages. Patterns of homogamy on such social characteristics as race, educational attainment, social class background, and religion are a key form of segregation.

Units of Analysis in Segregation Research

When analyzing segregation and other dynamic processes, one must determine the scale at which to model the process, and the related problem of determining the level of aggregation of the relevant components in time, space, or organization (Pascual 2001). In empirical work, these choices are often guided by practical considerations. Administrative boundaries, for example, may define neighborhoods for the researcher, but may not correspond to people's notions of neighborhoods or local communities (e.g., Grannis 1998). Similarly, analyses of marriage

patterns may define a marriage "market" as a census labor market area. One's inferences about segregation and segregation processes may depend on these assumptions.

Individuals may act alone or in concert. Marriages are (typically) two-person events, firms go through hiring cycles, and entire families change their in residences. A related consideration is the units of time in which behavior occurs. Smaller units of time reveal finer-grained transitions. A company may hire 10 employees in a month, but at most only one person per day. A neighborhood may lose 50 households per year, but only one or two households move in a week. The units of time in which we observe decisions may conceal important segregation processes. For example, if individual interactions are a key component of neighborhood change, gross annual population flows between neighborhoods may conceal important behavior. Similarly, if people choose homes first on the basis of housing-unit attributes and secondly on the basis of the neighborhood, correlations between population flows and the aggregate properties of housing stocks may conceal key behavioral mechanisms.

Segregation may result from overlapping and interrelated processes on multiple dimensions. Although residential patterns are governed by race, income, age, the presence of children, homeownership status and other dimensions, different dimensions of segregation may be more salient at different scales. Race and income may segregate individuals across neighborhoods, whereas religious affiliation, age, birthplace, and political beliefs may segregate individuals across larger regions. Neighborhoods may be relatively homogeneous in race but relatively heterogeneous in income, and cities may be relatively homogenous in political affiliation but heterogeneous in race.

Different segregation processes may play out on different scales. For example, the degree of job segregation by sex may differ substantially at the firm, occupation, or industry

levels (Bielby and Baron 1984). Similarly, explanations of segregation across neighborhoods may depend on individuals' ethnic tolerance and preferences for local amenities, whereas racial differentiation across cities may be caused by labor market processes, population composition, and political behavior. In addition, large and small populations or groups of locations may follow different dynamics. If human behavior is governed by random as well as systematic influences, lower levels of aggregation tend to exhibit more randomness than higher levels (e.g., small towns vs. large cities). For a fixed neighborhood size, changing the size of the population may affect segregation (Bruch and Mare 2006). The size of groups relative to population size may also affect the process. For example, the same level of ethnic tolerance or discriminatory behavior may produce low levels of segregation when we define neighborhoods as census tracts, but higher levels of segregation when we define neighborhoods as contiguous housing units.

Finally, it is necessary to specify the universe where segregating process occur. This includes the physical or spatial units along which we measure segregation (e.g., neighborhoods, married pairs, or occupations), the population at risk of segregation (e.g., all women between the ages of 25-49), and the spatial universe (e.g., Los Angeles County).

Markets and Other Institutions

For analytical simplicity, researchers often assume that individual actions, unconstrained by market imperfections, affect segregation. In fact, people are constrained by space, social organization, and time (e.g., social networks, career trajectories, or speed dating rules). In the housing market, for example, an individual's chances of hearing about a vacant housing unit may depend on its the distance from her current residence or on whether the it located where the neighbors are members of the individual's ethnic group or social class (e.g, Krysan 2006). Similarly, in a labor market a member of an ethnic minority may be relatively more likely to hear

about job openings at firms or in occupations where minorities are well represented (Montgomery 1991). More generally, organizational structures, such as internal labor markets, may channel access to mobility opportunities (e.g., Doeringer and Piore 1971). Barriers to communication or mobility among firms and the seniority rules that govern job trajectories may affect job and occupational segregation. Social organization and communication patterns may arise exogenously or may themselves be a function of segregation patterns and processes (Moody, this volume).

Models of Segregation Processes

We next consider some models for how people are allocated to positions through coordinated action. These models vary in their assumptions about the structure of positions and the key actors in the allocation process.

Queuing. In a simple queuing model, a heterogeneous population is available for entry into a set of social locations, such as a desirable jobs or occupations. Actors who control those locations, such as employers, rank potential employees by their task-related qualifications, such as education or prior experience, and their ascriptive traits, such as race or sex (e.g, Huffman and Cohen 2004). Employers hire top ranked individuals hire in sufficient numbers to fill available positions, forcing lower ranked individuals to take less desirable positions. Employers also rank their current employees and, during economic downturns, lay off workers who are at the bottom of that queue. This model can account for variations in group differences in unemployment and for the occupational segregation of groups defined by traits that affect rank in the queue. In this model employees are the only actors, and their actions are limited to following a queuing rule.

Vacancy chains. In vacancy models open positions, such as job or housing vacancies "move" within a hierarchical system (White 1970; Marullo 1985; Abbott 1990). Mobility

occurs when a position opens and the next individual "in line" for this position moves to fill it. Vacancy chains occur when the initial opening sets off a series of individual moves, possibly governed by a queuing mechanism, each filling the vacancy left by the previous occupant. The chain stops when the last position is either filled by a new entrant into the system or eliminated. Thus, opportunities for mobility depend on the availability of empty positions, and the mobility of any one person affects the opportunities available to others. In these models, the dynamics are governed by a fixed set of rules and positions. Although the mobility opportunities of individuals are interdependent, the model does not allow for coordinated individual choices.

Matching. Matching models assume that people are allocated to positions via markets, in which actors on both sides (e.g., buyers and sellers, men and women, etc.) compete for the most desirable match. The preferences and characteristics of actors on each side of the market simultaneously establish the opportunities of the other side. Gale and Shapley (1962) derived a matching equilibrium under the assumption that both sides of the market have full information about and ranked preferences for items available on the other side of the market. In this equilibrium, no actor on one side of the market can break his match and find a more preferred partner who also prefers him more than she prefers whomever she is already with. The likelihood that any man ends up with his preferred mate depends on not only his preferences and the preferences of his potential partner, but also on the preferences of all other actors in the market. Extensions of this model allow for incomplete information and sequential rather than simultaneous search processes (e.g., Mortenson 1978). Matching models represent market mechanisms that govern the allocation of people into jobs, houses, or interpersonal relationships and thus hold potential for understanding segregation processes. Use of these models in empirical work has been hampered by lack of suitable data in individuals' preferences and search

behaviors. Logan *et al.* (forthcoming) propose statistical models of two-sided choice that enable one to estimate the underling behavioral model using data only on existing matches (rather than complete data on individual preferences and search).

Location Choice. Models of *location choice* represent the effects of individual's preferences and choices on the distribution of individuals across social locations. In these models, individuals face a range of possible location choices (e.g., vacant dwellings, peer groups, job openings) in groups that have fixed characteristics in the short run. Their choices are governed by available vacancies and the costs and benefits of entering a location. In the longer run, the choices of individuals affect the opportunity structures of other individuals, not only by altering the vacancy distribution, but also by changing the characteristics of the locations they leave and enter. The result is a dynamic relationship between individual choices and the interdependence of mobility opportunities among individuals, but differ in allowing for change in the characteristics of these opportunities via shifting population composition. They share with matching models the coordinated choices of individuals, but differ in allowing for multi-sided rather than two-sided matching.

RESIDENTIAL SEGREGATION

Three key processes generate ethnic segregation: individuals' preferences for their own ethnic group (or avoidance of other groups) (e.g., Clark 1986), economic inequalities among ethnic groups combined with price differences in housing, and discrimination in such forms as redlining and racial steering by real-estate agents (e.g., Yinger 1995). Absent theoretical understanding of how neighborhoods change, it is difficult to identify the relative effects of these

several mechanisms. Much research on segregation trends is based on census-based crosssectional studies of cities (e.g., Massey and Denton 1993). Other studies, based on survey attitude data, investigate the willingness of individuals to live in neighborhoods of varying raceethnic composition (e.g., Farley *et al.* 1994). Still others audit the responses of real estate agents to potential home "buyers" with varying social characteristics who are in fact accomplices of the researcher (e.g., Turner *et al.* 2002). Although these data are informative, they do not directly show the dynamics of neighborhoods. Censuses do not show the intercensal behavior that governs trends in segregation. Individuals' stated preferences about neighborhoods do not alone explain residential patterns because they do not show their link to mobility behavior and aggregate segregation. Audit studies can demonstrate discrimination, but do not reveal how much segregation would occur in the absence of discrimination. Thus, models are needed to bridge the inference gap between processes at the individual, neighborhood, and city-wide levels of observation and analysis.

Measuring Residential Segregation

Summary measures of segregation establish the criteria for judging how well models of segregation processes explain aggregate residential patterns. Segregation is multidimensional in both concept and measurement. Massey and Denton (1988) identify five dimensions: *evenness*, the over/underrepresentation of groups in different parts of a city; *exposure*, contact between groups; *concentration*, confinement of one group in a small area; *centralization*, confinement of a group in the inner city; and *clustering*, residence in one contiguous space. Traditional segregation measures capture one or more of these dimensions. Measures of evenness include the index of dissimilarity, the Gini coefficient, and the entropy index (e.g., James and Taeuber 1985). Exposure is captured by isolation indices (James and Taueber 1985), whereas clustering

is measured by White's spatial proximity index (White 1983). Reardon and Firebaugh (2002) extend the two-group segregation measures to multigroup comparisons. For segregation of quantitative traits such as household income, researchers often rely on measures that are based on the relative sizes of between and within neighborhood variability (Jargowsky 1996).

Most commonly used segregation measures are *aspatial* in that they summarize the population composition of areas but do not take account of the spatial relationships among these areas. This may create a "checkerboard problem," in which a measure yields the same value where a racial minority is segregated into a single spatially contiguous region and as where minority neighborhoods are interspersed with white areas (White 1983). Reardon and colleagues (e.g., Reardon and O'Sullivan 2004; Reardon et al. 2006) develop spatial measures of segregation and show that, once distance between areas is taken into account, the five dimensions of segregation reduce to two: spatial exposure and spatial evenness. Their spatial segregation scores give more weight to the population of nearby locations than more remote locations. They also consider the role of scale in understanding segregation processes. Segregation may look different when examining lower or higher levels of aggregation (e.g., neighborhoods defined as the area surrounding a household with a radius of 500 versus 400 meters), and information about the degree to which segregation changes with scale may shed light on the underlying processes governing segregation. Segregation profiles show the level of segregation by scale in a given area, and the degree to which segregation changes with scale (e.g., blocks vs. census tracts).

Models of Residential Choice

We can formalize our discussion of the key elements in segregation processes, including the residential opportunity structures of individuals or families, their tastes and preferences, the

effects of their evolving knowledge and sentiments, and the links between the actions of individuals and the composition and segregation of neighborhoods (McFadden 1978). Although we refer to the choices of "individuals," the decision-makers may be other units such as families or agents acting on behalf of organizations such as firms or schools. We refer to "neighborhoods" but the model is potentially applicable to choices among specific dwelling units as well as larger community areas; and, with modification, to other types of social locations (jobs, church congregations, marriage partners, etc.) where segregation occurs.

Denote by U_{iit} the (latent) utility or attractiveness that the *i*th individual attaches to the *j*th neighborhood at time t. Let p_{ijt} denote the probability that the *i*th individual moves to the *j*th neighborhood at time t. The utility of a neighborhood for an individual depends on characteristics of the neighborhood and on the individual's own characteristics. To the analyst, these characteristics may be observed or unobserved, but these characteristics are known to the actors to whom they apply. Denote by Z_{jt} the observed characteristics of the *j*th neighborhood at time t (for example, the race-ethnic makeup of the neighborhood, the prices and rents of vacant houses, and the location of the neighborhood within a city, including distance from other neighborhoods with varying amenities and demographic characteristics. Let X_{it} denote observed characteristics of the *i*th individual at time *t*. These personal characteristics include fixed demographic characteristics such as race and sex and variable characteristics such as income as well as personal employment, income, family, and residential histories. Personal characteristics, including tastes and preferences may result from past residential experiences. Let η_{it} and ε_{it} be unobserved characteristics of neighborhoods and individuals respectively. Then the attractiveness of neighborhoods is:

$$U_{iit} = F(Z_{it}, X_{it}, \eta_{it}, \varepsilon_{it})$$
⁽¹⁾

If F as a linear random utility model, then, for example, for a single observed neighborhood and personal characteristic (Z and X respectively), the model is:

$$U_{ijt} = \beta Z_{jt} + \gamma X_{it} + \delta Z_{jt} X_{it} + \eta_{jt} + \varepsilon_{it} + \alpha Z_{jt} \varepsilon_{it} + \theta X_{it} \eta_{jt}$$
(2)

where β , γ , δ , α , and θ are parameters. This model incorporates a rich set of effects of observed and latent characteristics of persons and neighborhoods. When individuals choose where to live they implicitly compare neighborhoods in their choice set, that is, neighborhoods that they know about and where they may move with a nonzero probability. The comparison between the *j*th and the *j*'th neighborhood is

$$U_{ijt} - U_{ij't} = \beta_i^* (Z_{jt} - Z_{j't}) + \delta(Z_{jt} - Z_{j't}) X_{it} + \varphi_{jj't}$$
(3)

where $\beta_{ii}^{*} = \beta + \alpha \varepsilon_{ii}$ and $\varphi_{ij'i} = \eta_{ji} - \eta_{j'i}$. Utility differences between neighborhoods are a function of differences in their observed and unobserved characteristics (*Z* and η respectively). The characteristics of individuals do not affect the utility comparison additively because the comparison is within individuals, but they may interact with neighborhood characteristics. For example, the effect of differences in the proportion of persons in a neighborhood in a given ethnic group on the relative attractiveness of the neighborhoods is likely to differ between individuals who are members of that ethnic group and those who are not. Unmeasured characteristics of individuals may also modify the effects of neighborhood characteristics. These unmeasured characteristics (ε_{ii}) induce random variation in the β_i^* around the effects of measured neighborhood characteristics β . For example, the effect of the proportion of persons in the neighborhood who are ethnic minorities may depending on an individual's level of tolerance, which is unobserved to the analyst.

Given data on the characteristics of individuals and neighborhoods and the behaviors or stated preferences of individuals for neighborhoods, and an assumed probability distribution of

the unobserved characteristics of individuals and neighborhoods, it is possible to estimate the parameters of the discrete choice model. If the $\varphi_{jj't}$ follow an extreme value distribution, we obtain a *discrete choice* conditional logit model:

$$p_{ijt}(Z_{jt}, X_{it}, C_{(i)}) = \frac{\exp(\beta_i^* Z_{jt} + \delta Z_{jt} X_{it} + \eta_{jt})}{\sum_{k \in C_{(i)}} \exp(\beta_i^* Z_{kt} + \delta Z_{kt} X_{it} + \eta_{kt})}$$
(4)

where $C_{(i)}$ denotes the set of neighborhoods available ("choice set") for the *i*th individual which may be restricted to incorporate discrimination, prices, or information constraints. For example, the choice set may be restricted to units within a given radius of a person's current home, to units in neighborhoods that are at least 10 percent own-race, or to units where monthly rent or mortgage payments would be less than some fraction of individuals' incomes. Although this model is a variant of standard choice models for consumer demand, it is possible to extend it so as to take account of special features of residential choice and mobility. Some of these extensions are as follows:

Price Effects. Prices reflect aggregate housing demand and may be a positive function of the expected number of people who seek to move into each neighborhood. Lower prices offset the unattractive traits of some neighborhoods and thus attract residents. Housing markets "clear" when prices adjust such that for any unit of time the expected number of households moving into any dwelling unit is 1 (if the number of dwellings is less than or equal to the number of households) or the ratio of households to housing units (if dwellings exceed households).

The Choice Not to Move. Individuals may evaluate their current location differently from other possible destinations. For example, whites may tolerate a higher level of racial integration in their current neighborhood, because neighbors are already known. Alternatively, people may assess neighborhood characteristics in their current location in the same way that they respond to

possible destination areas, but moving costs deter mobility and make the current residence more attractive, *ceteris paribus*.

The Effects of Neighborhood Change and Individual Experience. Individuals may respond, not only to the current characteristics of neighborhoods, but also to how those characteristics have changed in the recent past. People may use past information about neighborhood change to forecast future neighborhood change, a phenomenon reflected in popular parlance that "the neighborhood is changing." Additionally, individuals' own residential histories may change their preferences. People who have lived in integrated neighborhoods, for example, may be more or less tolerant of diversity.

Supply of Housing. Whether an individual can move into a housing unit or geographic area depends on the supply of vacant housing. If the discrete choice model applies to specific dwellings, then the choice set consists of available (vacant or soon to be vacant) units and mirrors the supply of housing. However, if the choice model applies to aggregations of dwelling units such as neighborhoods, then it is necessary to take an indirect approach to housing supply by including the log of the number of vacancies or the number of dwelling units as a regressor in equation (4) (McFadden 1978).

Some Specific Behavioral Models

Using the general model of location choice we can examine specific behavioral rules. A variant of Schelling's notion that people are willing to live in neighborhoods as long as they are the local majority yields the model:

$$p_{ijt}(Z_{jt}) = \frac{\exp(1^{*}\{Z_{jt} \ge 0.5\})}{\sum_{k \in C_{(i)}} \exp(1^{*}\{Z_{jt} \ge 0.5\})}$$
(5)

where Z_{ii} is the proportion own-race in neighborhood *j*, and 1^{*} {} is an indicator function that

equals 1 if the expression in brackets is true, and 0 otherwise. This threshold function assumes that people only differentiate between neighborhoods above and below 50 percent own group and are indifferent to compositional differences among neighborhoods that do not cross the threshold. Alternatively, individuals may prefer to be the local majority, but respond to small changes in neighborhood composition, yielding a continuous function in the proportion within an ethnic group:

$$p_{ijt}(Z_{jt}) = \frac{\exp(Z_{jt})}{\sum_{k \in C_{(i)}} \exp(Z_{kt})}.$$
(6)

Equations (5) and (6) are illustrated in Figure 1. These two functions have the same average level of tolerance, but make different assumptions how people evaluate neighborhoods. These and other specifications can be tested for their adequacy in fitting individual data on residential choice and investigated for the type of aggregate segregation patterns that they imply.

Strategies for Linking Residential Mobility and Neighborhood Change

Three related strategies for linking data at the individual and neighborhood level to test hypotheses about the causes of segregation are Markov models, general equilibrium models with prices, and agent-based models. Each of these models blends individuals' neighborhood choices with aggregate neighborhood change. Given individual preferences and initial neighborhood conditions, these approaches treat both the demographic composition of neighborhoods and mobility behavior as endogenous. Whereas Markov models are a form of macrosimulation (mobility occurs through expected rates of transition), agent-based modeling is a form of microsimulation (individuals move according to stochastic realizations of probabilistic choice) (Macy and Flache, this volume). The general equilibrium approach relies on expected rates of mobility, but is computed from individual-level cross section data.

These strategies combine data at the individual and aggregate levels, combining estimated variants of the discrete choice models from survey data, observations of neighborhood conditions from census or other administrative data, and estimated probabilities of mobility between neighborhoods. Administrative data define the initial population distribution, as well as fixed attributes of neighborhoods. The models generate subsequent and equilibrium neighborhood distributions implied by a set of individual choice probabilities. The model equilibrium is a distribution of individuals across neighborhoods such that no person can expect to improve his situation, given the decisions of all other individuals. For a wide range of residential choice functions, there are one or more population level equilibria (e.g., Brock and Durlauf 2001). Equilibria may be static (the expected distribution of people across neighborhoods is unchanging) or dynamic (the distribution of people across neighborhoods cycles through a finite set of states). They may be unique (only one distribution of individuals across neighborhoods is an equilibrium) or non-unique (individual behavior implies multiple possible stable distributions). They may be robust to small random perturbations or, alternatively, small departures from equilibrium may cumulate, ultimately moving the system into a new equilibrium.

Markov Models. Markov models link a set of individual- or group-specific residential mobility probabilities to expected patterns of neighborhood turnover. A Markov model has a finite set of *K* states, $S = \{s_1, s_2, ..., s_k\}$. The states can be specific neighborhoods (for example, Census tracts in a city) or neighborhood types (for example, poor vs. nonpoor neighborhoods). The expected distribution of the population across the *K* states at time *t*, is

$$m[t] = [m_1^1(t), ..., m_{\kappa}^1(t), m_1^2(t), ..., m_{\kappa}^2(t), ..., m_1^G(t), ..., m_{\kappa}^G(t)],$$

where superscript g = 1, 2, ..., G indexes group membership (e.g., ethnic groups). We also specify

a *GK* by *GK* matrix **P** of conditional probabilities that a member of group *g* moves to state *j* at time t + 1 conditional on being in state *i* at time *t*. Markov models assume that the distribution of the population at time t+1 depends only on characteristics and locations of the population at time *t* (and no prior time periods). The population distribution at time t+1 is therefore

$$m[t+1] = \mathbf{P}m[t]. \tag{7}$$

Markov models usually assume time-invariant constant probabilities (P) of moving between state (e.g., Quillian 1999). However, if individuals react to and transform their neighborhoods through their mobility behavior, then their behavior follows an *interactive* Markov model (Conlisk 1976) where the elements of **P** depend on the population distribution at time *t*:

$$m[t+1] = \mathbf{P}(m[t])m[t]. \tag{8}$$

If, for example, m[t] represents the distribution of blacks and whites across neighborhoods, then the probability of moving into a given neighborhood is a function of its race composition. In this model, preferences for neighborhood characteristics are fixed, but the attractiveness of specific neighborhoods changes as a result of their changing characteristics. (In principle, the same model may be represented as a fixed rate Markov model of mobility between neighborhood *types*, but an interactive Markov model of mobility between specific neighborhoods.)

Mare and Bruch (2003) investigate residential segregation using an interactive Markov models. The transition matrix **P** is based on rates of residential mobility estimated from longitudinal survey data. Each iteration of the model represents a unit of time and, with sufficient iterations, the neighborhoods reach an equilibrium distribution. The equilibrium, however, results from the simplified conditions represented in the model. Actual cities seldom reach an equilibrium distribution because they are subject to external shocks (for example, new housing construction or immigration).

A Markov models predicts the implications of a given profile of behavior for a specified state space. The state space is typically neighborhoods or neighborhood types, and the population moving between states is typically described by a limited number of discrete characteristics (for example, black/white and poor/non-poor). Although the state space may, in principle, include an arbitrary number of dimensions of individual and neighborhood characteristics, multidimensional models are unwieldy because the state space increases dramatically with each added dimension (van Emhoff and Post 1998). Microsimulation approaches such as agent-based modeling may be a more useful method in these cases.

General Equilibrium Models with Price Effects. A second approach to linking individual behavior with population dynamics is through general equilibrium models with prices. This approach allows the entire population to adjust to the new environment, thereby changing the makeup of neighborhoods. Bayer and colleagues (e.g., Bayer, McMillan, and Rueben 2004) apply this framework to examine the degree to which ethnic preferences affect patterns of ethnic segregation. Their approach consists of a model of residential choice at the individual level and a simulation of the implications of that model for patterns of neighborhood change. It includes a discrete choice model for the probability of choosing a housing unit, conditional on its price and other attributes (e.g., owned versus rented, number of rooms, elevation) and neighborhood characteristics (e.g., air quality, ethnic composition, average household income). Some housing unit and neighborhood attributes, such as size and air quality are fixed whereas others, such as neighborhood ethnic composition, are endogenous to the sorting process. Each household optimizes its location given the set of alternatives and the decisions of other households (thus, the parameters are estimated assuming the distribution of households is in equilibrium).

The estimated behavioral model is a tool for examining how neighborhoods achieve different equilibria depending on variation in preferences or population composition. A change in preferences or household characteristics (e.g., an elimination of race preferences) yields a new equilibrium distribution of households across neighborhoods via the mechanisms of priceregulated changes in supply and demand for housing. The model adjusts housing prices so that markets clear; that is, the expected number of households moving into each housing unit is 1. The model (1) generates neighborhood housing prices that clear the market; (2) given new prices, predicts the probability that each household chooses each housing unit; (3) aggregates housing choice probabilities at the neighborhood level to estimate the revised socioeconomic composition of each neighborhood; (4) estimates the new socioeconomic composition of each neighborhood; and (5) repeat steps 1-4 until convergence. Unlike Markov models, which show how neighborhoods change en route to equilibrium, the general equilibrium approach compares old and new equilibria following a change in model inputs. The latter approach assumes equilibrium at the beginning of the simulation, whereas Markov models do not make this assumption.

Agent-based models represent neighborhood change as the result of the interdependent decisions of individuals ("agents") who follow specified rules of behavior. For example, agents obtain information about neighborhoods and decide whether to move to a new housing unit or stay in their current housing unit. Each agent's decision changes the social landscape for other agents, thus affecting their subsequent behavior. Agents' decisions about where to move are individual-level stochastic realizations of transition probabilities governed by a discrete choice model (5). Given the characteristics of the agent and the housing vacancies to which it is exposed, the agent's decision is determined by a random draw from the probability distribution

generated by the discrete choice model (Bruch and Mare 2006). Because agent models represent all information at the individual level, including an individual's complete residential history, they allow for a more flexible specification of the state space and more complex interactions between an agent and its environment than Markov models (van Emhoff and Post 1998).

Schelling (1971, 1978) used a simple agent-based model for residential segregation in which two groups ("blacks" and "whites") are distributed across a grid in accordance with their preferences about neighborhood racial makeup. Each agent wishes to live where at least 50 percent of its neighbors are members of its own group. Irrespective of how agents are initially distributed, they try to move whenever they are surrounded by a majority of their own color. This model predicts a resulting pattern of segregation more severe than any individual alone would prefer. Zhang (2003) and Pancs and Vriend (2007) show that that ethnic segregation occurs even when agents prefer to live in an integrated neighborhood. Bruch and Mare (2006) simulate the segregation dynamics when individuals have the same average tolerance as Schelling's threshold, but differ in how they respond to small changes in neighborhood composition (Figure 1). Higher levels of segregation occur when agents' preferences follow a threshold function. When agents make finer-grained distinctions among neighborhoods that vary in racial composition, preferences alone may not lead to segregation because the segregating effects of changes in the size of the population at risk of entering a neighborhood are offset by a corresponding change in that neighborhood's desirability. Other studies link Schelling's behavioral model to empirical studies of racial tolerance and residential choice (Bruch and Mare 2006; Fossett 2006) or more realistic geography and population composition (Benenson 2004; Bruch 2006).

Agent-based models are also valuable for exploring the sensitivity of conclusions about

segregation to alternative assumptions about individual behavior, neighborhood size, and population size. Holding the size of agents' neighborhoods fixed, the same preference function can produce a low level of segregation in a large population, but a high level of segregation in a small population. In a small population, when agents evaluate neighborhoods according to a continuous function, changes in the size of the population at risk of entering a neighborhood may be too large to offset the change in that neighborhood's desirability (Bruch and Mare 2006).

MULTIPLE SEGREGATION PROCESSES

Notwithstanding the focus of this essay on racial residential segregation, people sort on many factors, including age, income, and lifestyle, and may be segregated across multiple sets of locations, including schools, workplaces, families, and others. Insofar as characteristics of individuals or households are correlated, processes that increase segregation in one grouping may reinforce or attenuate segregation across other groupings. Similarly, residential segregation may reinforce or attenuate segregation across other sets of social locations.

Residential Segregation by Race and Income

We illustrate the process of segregation on two dimensions, race and income. Figure 2 illustrates a possible relationship between housing costs and the probability of choosing a dwelling. Suppose that the probability of moving into a given housing unit increases with the unit price up to a threshold c*. People want to live in the best housing they can afford, but, by definition cannot live at all in unaffordable housing. Thus, they distinguish among affordable but not unaffordable dwellings. Given a level of racial tolerance and income inequality among races, sorting by income may exacerbate or attenuate ethnic segregation. The distributions of race and income together can produce more *or* less segregation than race alone (depending on

the ethnic group). For example, in Los Angeles, whites are less tolerant toward blacks than other ethnic groups, but Hispanics have on average substantially lower incomes than either blacks or whites. As a result, whites live closer to blacks than their ethnic preferences alone would imply (Bruch 2006). Similarly, a reduction in income disparities between blacks and whites may *increase* race segregation because an enlarged black middle-class may sustain segregated black middle-class neighborhoods that would not otherwise be possible (Sethi and Somonathan 2004; Bayer, Fong, and McMillan 2006).

Overlapping Social Locations

Segregation also occurs across overlapping sets of social locations. Residential segregation may, for example, be interdependent with marriage markets and patterns of assortative mating, friendship networks in high schools, job referral networks, and occupational segregation. These connections arise because of the social ties created by both opportunities for contact provided by physical propinquity and the creation of normative expectations.

Residential Segregation and Marriage. Although people may meet their future partners in a variety of locations (Kalmijn 2001), the neighborhood is a key context in which they form romantic ties. Proximity to potential partners with similar social characteristics contributes to marital homogamy on those characteristics. Norms about inter-group contact may depend on opportunities for contact among different groups and patterns of assortative mating. For example, if people living in educationally integrated neighborhoods may be less likely to form relationships and, if they do marry, are more likely to marry heterogamously. Conversely, educationally homogenous neighborhoods tend to narrow the range of potential partners that an individual meets. Single persons in homogeneous neighborhoods may be more likely to marry and, when they marry, to marry homogamously. (Married couples in homogenous neighborhoods, however, may also be more likely to separate, given high rates of contacts with other desirable mates). Single persons may move to neighborhoods where there are people who are demographically similar to themselves, thereby reinforcing residential segregation along demographic lines. Once married, a homogamous couple may also be more likely to reside in a neighborhood that is demographically homogeneous and similar to the couple themselves - more so than if the couple is heterogamous and is forced to choose a heterogeneous neighborhood or one that resembles one spouse more than the other. In short, the combined marriage choices and residential decisions of single and married persons may mutually reinforce marital homogamy and residential homogeneity. Of course, some people do marry heterogamously and live in neighborhoods with persons demographically different from themselves. When this behavior occurs often enough, it may offset the mutually reinforcing segregative effects of marriage and residential patterns. Heterogeneous neighborhoods provide, through both propinquity and norm formation, a catalyst for heterogamous marriages, and heterogamous couples may contribute to desegregation. Which of these competing effects dominate the marriage and housing markets depends on the strength of marital and residential preferences. Their dynamics is a fertile ground for segregation research.

Residential Segregation, School Segregation, and Friendships. Because children are assigned to school districts based on where they live and parents choose neighborhoods in part on the basis of the quality of available schools, residential and school segregation are closely linked. Because schools foster friendships, neighborhood segregation may affect the segregation of children into ethnically or economically homogeneous peer groups. Thus residential segregation may affect children's friendships and peer groups both directly because of geographic propinquity and indirectly through school segregation and its effects on children's

environments (e.g., Mouw and Entwisle 2006). However whereas attending a racially diverse school may increase opportunities for contact, it may also increase ethnic tension and therefore decrease the likelihood of forming a bond conditional on contact. Thus residential integration can reduce peer segregation by increasing opportunities for interracial contact, but increase peer segregation by decreasing the likelihood of friendship conditional on contact.

Residential and Workplace Segregation. Residential segregation may affect workplace segregation and unemployment in two ways. First, neighborhood segregation places disadvantaged populations further from job opportunities, thus increasing job search costs and commuting times (Holtzer 1991). Physical distance between work and home may affect employment outcomes. Second, residential segregation may isolate disadvantaged populations from social networks that lead to job referrals, regardless of the physical location of the job (e.g., Mouw 2002). Both mechanisms mutually reinforce residential and occupational segregation. If individuals cannot obtain well paying jobs, they are less likely to be able to afford to move into neighborhoods more proximate to better employment opportunities, further reinforcing both physical separation along socioeconomic lines and differential access to information about jobs.

CONCLUSION

The models and methods required for advancing knowledge about segregation processes are well within the sights of researchers working in the field and much can be learned by carrying out empirical work using the analytic strategies discussed here. Yet our knowledge of segregation processes remains in its infancy. We can identify many of the behaviors and environments that may influence levels and trends in segregation in a variety of empirical contexts. But much more research is needed to show which factors matter most in various

empirical applications or to judge the sizes of their effects. This research requires painstaking and creative attention to model refinement at both micro and macro levels along with rigorous empirical validation every step of the way.

REFERENCES

- Abbott, Andrew. 1990. "Vacancy Models for Historical Data." Pp. 80-102 in *Social Mobility and Social Structure*, edited by R. Breiger. New York: Cambridge University Press.
- Bayer, Patrick, Robert McMillan, and Kim Rueben. 2004. "Residential Segregation in General Equilibrium." NBER Working Paper No. 11095.
- Bayer, Patrick, Hamming Fang, and Robert McMillan. 2006. "Separate When Equal? Racial Inequality and Residential Segregation." NBER Working Paper 11507.
- Benabou, Roland. 1996. "Heterogeneity, Stratification, and Growth: Macroeconomic Implications of Community Structure and Educational Finance." *American Economic Review* 86:584-609.
- Benenson, Itzhak. 2004. "Agent Based Modeling: From Individual Residential Choice to Urban Residential Dynamics." Pp. 67–95 in *Spatially Integrated Social Science*, edited by Michael Goodchild and Donald Janelle. New York: Oxford University Press.
- Bielby, William and James Baron. 1984. "A Woman's Place is with Other Women: Sex Segregation within Organizations." Pp. 27-55
- Brock, William and Steven Durlauf. 2001. "Discrete Choice with Social Interactions." *Review of Economic Studies*. 68:235-60.
- Bruch, Elizabeth E. and Robert D. Mare. 2006. "Neighborhood Choice and Neighborhood Change." *American Journal of Sociology* 112:667-709.
- Bruch, Elizabeth E. 2006. "Dynamic Models of Ethnic and Economic Segregation." Unpublished Dissertation.
- Charles, Maria, and David B. Grusky. 2004. *Occupational Ghettos*. Stanford, CA: Stanford University Press.
- Clark, William. 1986. "Residential Segregation in American Cities: A Review and Reinterpretation." *Population Research and Policy Review* 5:95-127.
- Conlisk, John. 1976. "Interactive Markov Chains." Journal of Mathematical Sociology 4:157-85.
- Doeringer, Peter and Michael Piore. 1971. *Internal Labor Markets and Manpower Analysis*. Lexington, MA: Health Lexington Books.
- Durlauf, Steven. 1996. "A Theory of Persistent Income Inequality." *Journal of Economic Growth* 1:75-93.

- Farley, Reynolds, Charlotte Steeh, Maria Krysan, Keith Reeves, and Tara Jackson. 1994. "Segregation and Stereotypes: Housing in the Detroit Metropolitan Area." *American Journal of Sociology* 100:750–80.
- Fossett, Mark. 2006. "Ethnic Preferences, Social Distance Dynamics, and Residential Segregation: Theoretical Explorations Using Simulation Analysis." *Journal of Mathematical Sociology* 30:185-273.
- Grannis, Rick. 1998. "The Importance of Trivial Streets: Residential Streets and Residential Segregation." *American Journal of Sociology* 103:1530-1564.
- Hernes, Gutmund. 1972. "The Process of Entry into First Marriage." *American Sociological Review* 37:173-82.
- Holzer, Harry. 1991. "The Spatial Mismatch Hypothesis: What Has the Evidence Shown." Urban Studies 28:105-22.
- Huffman, Matt and Philip Cohen. 2004. "Racial Wage Inequality: Job Segregation and Devaluation across U.S. Labor Markets." *American Journal of Sociology* 109:902-1036.
- James, David and Karl Taueber, 1985. "Measures of Segregation." Pp. 1-32 in *Sociological Methodology* 15:1-32.
- Jargowsky, Paul. 1996. "Take the Money and Run: Economic Segregation in U.S. Metropolitan Areas." *American Sociological Review* 61:984-98.
- Kalleberg, Arne, Michael Wallace, and Robert Althauser. 1981. "Economic Segmentation, Worker Power, and Income Inequality." *American Journal of Sociology* 87:651-83.
- Kalmijn, Matthijs. 1998. "Intermarriage and Homogamy: Causes, Patterns, Trends." *Annual Review* of Sociology 24:395–421.
- Kalmijn, Matthijs, and H. Flap. 2001. 'Assortative Meeting and Mating: Unintended Consequences of Organized Settings for Partner Choices.' *Social Forces*, 79:1289-1312.
- Krysan, Maria. 2006. "Does Race Matter in the Search for Housing? Search Strategies, Locations, and Experiences." Paper presented at the annual meetings of the Population Association of America. Los Angeles, CA.
- Levin, Simon. 1992. "The Problems of Pattern and Scale in Ecology: The Robert H. MacArthus Award Lecture." *Ecology* 6:1943-1967.
- Logan, John, Peter Hoff, and Michael Newton. 2007. "Two-sided Estimation of Mate Preferences for Similarities in Age, Education, and Religion." Forthcoming in the *Journal of the American Statistical Association*.

- Mare, Robert and Elizabeth Bruch. 2003. "Spatial Inequality, Neighborhood Mobility, and Residential Segregation." California Center for Population Research Working Paper No. 003-03. University of California, Los Angeles.
- Marullo, Sam. 1985. "Housing Opportunities and Vacancy Chains." Urban Affairs Quarterly 20:364-88.
- Massey, Douglas and Nancy Denton. 1988. "The Dimensions of Residential Segregation." *Social Forces* 67:281-315.
- Massey, Douglas, and Nancy Denton. 1993. *American Apartheid: Segregation and the Making of the Underclass*. Cambridge, Mass.: Harvard University Press.
- McFadden, Daniel . 1978. "Modelling the Choice of Residential Location." Pp. 75-96 in A. Karlqvist, L. Lundqvist, F. Snickars, and J. Weibull (eds.), *Spatial Interaction Theory* andPlanning Models. Amsterdam: North-Holland.
- Montgomery, James. 1991. "Social Networks and Labor Market Outcomes: Toward an Economic Analysis." *The American Economic Review* 81:1408-18.
- Mortensen, Dale. 1988. "Matching: Finding a Partner for Life or Otherwise." *American Journal of Sociology* 94:S215-S240.
- Mouw, Ted. 2002. "Are Black Works Missing the Connection? The Effect of Spatial Distance and Employee Referrals on Interfirm Racial Segregation." *Demography* 39:507-28.
- Mouw, Ted and Barbara Entwisle. 2006. "Residential Segregation and Interracial Friendship in Schools." *American Journal of Sociology* 112:394-441.
- Pancs, Romans and Nicolaas Vriend. 2007. "Schelling's Spatial Proximity Model of Segregation Revisited." *Journal of Public Economics* 1:1-24.
- Pascual, Mercedes. 2001. "Scales that Matter: Untangling Complexity in Ecological Systems." Pp. 255-86 in *Carving our Destiny: Scientific Research Races a New Millennium*. National Academy of Science and Joseph Henry Press.
- Quillian, Lincoln. 1999. "Migration Patterns and the Growth of High-Poverty Neighborhoods, 1970-1990." *American Journal of Sociology* 105:1-37.
- Reardon, Sean and Glenn Firebaugh. 2002. "Measures of Multigroup Segregation." *Sociological Methodology* 32:33-67.
- Reardon, Sean and David O'Sullivan. 2004. "Measures of Spatial Segregation." *Sociological Methodology* 34:121-62.

- Reardon, Sean, Stephen Matthews, David O'Sullivan, Barry Lee, Glenn Firebaugh, and Chad Farrell. 2006. "Segregation and Scale: The use and interpretation of spatial segregation profiles for investigating the causes, patterns and consequences of residential segregation." Paper presented at the 2006 meetings of the Population Association of America, New York.
- Reskin, Barbara and Patricia Roos. 1990. Job Queues, Gender Queues: Explaining Women's Inroads into Male Occupations. Philadelphia: Temple University Press.
- Roth, Alvin E. and Marilda Sotomayor. 1990. *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis.* Cambridge: Cambridge University Press.
- Schelling, Thomas. 1971. "Dynamic Models of Segregation." *Journal of Mathematical Sociology* 1:143–86.
- ——. 1978. *Micromotives and Macrobehavior*. New York: Norton and Company.
- Schwartz, Christine R. and Robert D. Mare. 2003. "The Effects of Marriage, Marital Dissolution, and Educational Upgrading on Educational Assortative Mating." CCPR Working Paper 026- 03. Under revision.
- Sethi, Rajiv and Rohini Somanthan. "Inequality and Segregation." *Journal of Political Economy* 112:1296-1321.
- Turner, Margery, Stephen Ross, George Galster, and John Yinger. 2002. Discrimination in Metropolitan Housing Markets: Phase I – National Results from Phase I of the HDS 2000.
 Washington DC: US Department of Housing and Urban Development.
- Van Imhoff, Evert and Wendy Post. 1998. "Microsimulation Methods for Population Projection." *Population: An English Selection* 10:97-138.
- White, Harrison. 1970. Chains of Opportunity. Cambridge, MA.: Harvard University Press.
- White, Michael. 1983. "The Measurement of Spatial Segregation." *American Journal of Sociology* 88:1008-18.
- Wilson, William J. 1987. The Truly Disadvantaged. Chicago: University of Chicago Press.
- Yinger, John. 1995. Closed Doors, Opportunities Lost: The Continuous Costs of Housing Discrimination. New York: Russell Sage.
- Zhang, Junfu. 2004. "A Dynamic Model of Residential Segregation." *Journal of Mathematical Sociology* 28:147-170.

Figure 1. Two Hypothetical Decision Rules



Figure 2. Hypothetical relationship between housing costs and household resources



Ratio of Housing Costs to Household Income

Figure 2.